A Unified Analysis and Comparison of Long PN-Code Acquisition Techniques for AWGN and Multipath Fading Channels

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ABSTRACT:
Direct Sequence Spread Spectrum is a solution for providing security and jamming resistance in wireless communications. Using long spreading codes is a good idea for making DSSS more secure and reliable. In this paper, common methods of long code acquisition are reviewed and discussed analytically and numerically. The effect of residual frequency and coherent integration time on their probability of detection and mean acquisition time in an AWGN channel are simulated and computational burden of each algorithm is roughly obtained. From theoretical point of view, in addition to restating the expressions of the methods in uniform framework for AWGN channel, performance of averaging based methods in AWGN channel are specifically analyzed. Also the effect of multipath Rayleigh channel on detection performance of the methods are modelled mathematically and simulated. Beside good agreement between theory and simulation, the results show that Dual folding method provide better trade-off between time and detection performance and passing through a typical fading channel causes a few dBs drop in the performance of the methods. This paper provides a comparison between acquisition techniques of long codes so as choosing a method for a specific application while considering trade-offs between metrics would be feasible.

KEYWORDS: Direct Sequence Spread Spectrum (DSSS), Code acquisition, Long PN-code, Multipath Fading, Detection Probability, Doppler frequency.

1 INTRODUCTION

Satellite communications often use line of sight wireless connections; on this type of channel corruptions exist constantly and interfere with the communication as thermal noise, fading, intentional jamming, spoofing and eavesdropping. Spread spectrum technique is an efficient way of tackling these problems by increasing the processing gain of the transfer link. Direct sequence spread spectrum (DSSS) achieves this by multiplying a spreading code directly with the modulated data [1]. Using common short period spreading codes doesn't provide good security since the code is easy to guess for an unauthorized user of positioning signals or an interceptor of a peer to peer link; On the contrary long codes with a period even longer than the message are unlikely to be found. Also long code achieves higher tolerance for spoofing and jamming [2]. However the acquisition is different from that of short codes' due to the larger uncertainty region of code phases to be searched. Two instances of long code applications are in Global Positioning System (GPS) as precision code (P-code) which provides accurate positioning [3, 4] and inter-satellite links for GPS and BeiDou Navigation Satellite System (BDS) to relay data without an intermediate ground segment [5, 6]. Regarding these applications the transfer link discussed in this paper is line of sight with probable multipath fading near the ground (if the receiver is positioned in dense areas).

Long code acquisition methods are usually based on hybrid search, which tests a block of code phases at a time in frequency domain using fast Fourier transform (FFT) techniques and tests sequent blocks the same way one after another. Some of these methods exhibit remarkable performance in terms of either probability of detection or mean acquisition time (MAT); the two main criteria often used for comparison between acquisition techniques. In [7] and [8] extended replica folding acquisition search technique (XFAST) is developed. XFAST estimates the time and frequency ambiguity by correlating the locally generated folded blocks with the received code blocks. Considering folding number as M, in this method the uncertainty region would be $1/M$ times smaller which clearly

تاریخ دریافت: ۱۳۹۷/۰۸/۲۲ تاریخ پذیرش: ۱۳۹۸/۰۵/۲۸

accelerates the searching process, but the performance is weak in low signal to noise ratios (SNR). In [9] Dual Folding (DF) method is proposed to compensate for this deficiency in XFAST. DF folds both incoming samples and local code samples and improves detection performance in low SNRs by increasing the coherent integration time. DF method with proper choice of the corresponding parameters can even precede Zero Padding (ZP) method's detection probability while searching the uncertainty region with much higher speed [9]. ZP is a hybrid search method that pads zeros to incoming samples to extend the length before taking FFT. This enables ZP to search some phases in parallel [10]. In [10] to enhance the performance of ZP, Generalized ZP (GZP) is proposed. GZP optimizes the length of zeros padded to incoming samples according to SNR and thereby presents a better trade-off between detection performance and parallel searching capability compared to ZP method. Another way for increasing parallel searching capability is averaging based methods like Direct Averaging Method (DAM) and Overlap Average Method (OAM) [11-13]. DAM reduces uncertainty region by averaging samples in local code and received signal, the rest of the method is similar to ZP or GZP. OAM uses modified averaging to improve performance of DAM.

Some other types of acquisition methods estimate the initial states of the shift registers which generate long spreading code. These methods use iterative message passing algorithms as in channel decoding [14-16]. However, the application of these methods are limited to few types of PN codes like maximum length sequences and gold codes, where a Tanner graph can be developed.

The mentioned methods are analyzed in the references and expressions for detection probability and MAT are developed and partly compared to each other. But these methods haven't been expressed in a unified framework where they can be better understood and compared and also no comprehensive simulations are provided to compare these methods with each other. Meanwhile, though the performance of averaging methods is examined theoretically in mentioned references but the process are vague and expressions are ambiguous.

In this paper a comparison between mentioned methods are presented, in terms of theoretical equations and simulation results. The methods are simulated considering the effect of residual frequency (mainly due to Doppler Effect) and coherent integration time in terms of probability of detection and mean acquisition time in an Additive White Gaussian Noise (AWGN) channel. While DF and GZP methods were previously examined extensively, here for completing long code acquisition techniques analysis, averaging methods of DAM and OAM are mathematically modelled and theoretical expressions for probability of detection and mean acquisition time are derived descriptively which is different from the general analysis already presented in [12] and follows another way to derive detection probability and MAT. The performance of the methods are also examined in multipath fading channel and theoretically expressed then followed by simulations. In addition, computational burden of each algorithm is roughly obtained to provide a comparison regarding resource consumption.

Next section reviews common methods of long code acquisition, their merits and flaws in comparison to each other. In section 3 and 4 detection performance of named methods is analyzed and theoretical expressions are derived for AWGN and multipath Rayleigh channels. Mean acquisition time of the methods is analyzed in section 5. Also proposed analysis of averaging methods is brought in these sections. Section 6 includes the analytical and numerical results of each methods performance for some of the effective parameters like Doppler. Section 7 concludes the paper.

2 LONG CODE ACQUISITION METHODS

The basis of long code acquisition methods is correlation in frequency domain which speeds up each code phase cell/frequency bin searching process. The acquisition time gets challenging when the uncertainty region is wide i.e. the spreading code's period is long. There are some leading acquisition methods that offer specific trade-off between detection performance and parallel search capability. Whether SNR is the restriction or hardware processing power and speed or other limitations, one of these methods come useful. These methods can be divided into three groups; zero padding, folding and averaging.

Figure 1 shows the process of acquisition in a long code direct sequence receiver. The specific algorithm of each of the following methods is applied on digital IF signal and after Analog to Digital Converter (A/D), just before correlation with FFT. The inevitable problem in long code acquisition resulting in detection performance degradation is the aperiodic partial correlation that happens in long code acquisition and causes self-noise in the results. Another challenge is the prolonged acquisition time that is caused by wide uncertainty region of a long code to be searched. As said, this can be dealt with by correlation in the frequency domain. But blocks chosen for correlation should have some overlapping to count for cases that the offset lies somewhere in the middle of a block, thereon GZP method which is based on ZP method is proposed which pads zeros to the samples of the received signal to enable correlating two signals with unequal lengths. However there's no consideration for long acquisition time caused by wide uncertainty region of long codes in GZP method. Using this method, $N-L$ samples of received digital IF signal are taken, where N is the FFT length then L zeros are padded to signal's end and the length reaches to N.

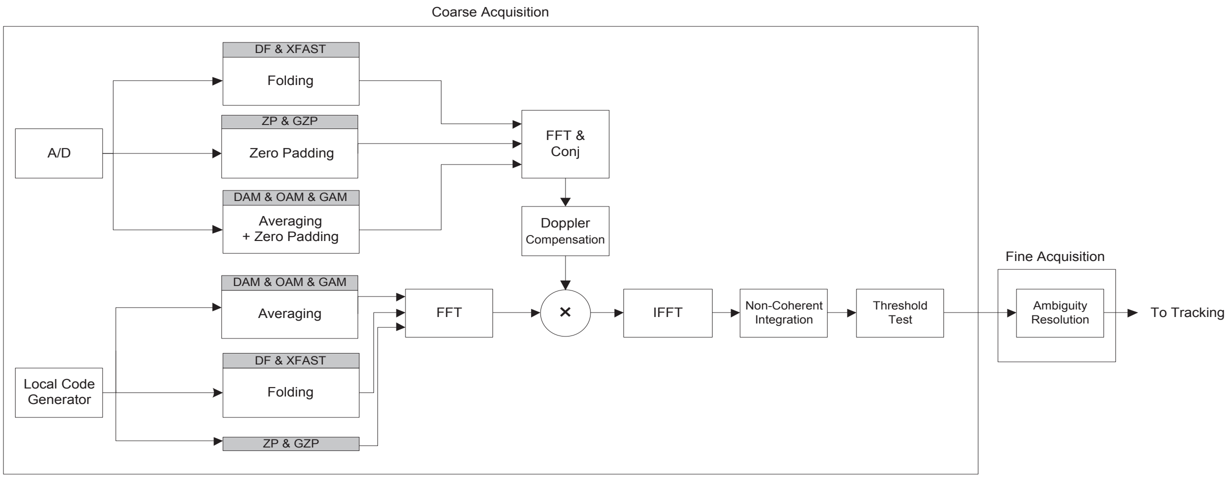

Fig. 1. Acquisition in a long code DSSS system

Conjugated FFT of these samples is multiplied with FFT of N samples of locally generated spreading code. Taking IFFT yields the correlation results. The first $L+1$ elements are searched for a maximum passing a threshold, if so the code phase would be the place of maximum result otherwise the local code is shifted $L+1$ samples and the search process is restarted. In ZP method, L is $N/2$ samples, while in GZP method L is chosen relative to incoming SNR. In the corresponding paper, by considering Doppler frequency and desired operational SNR, optimum length of padded zeros can be obtained. GZP method however has a poor parallel searching capability of $L+1$ phases meaning that in every correlation these phases are processed in the same time, which isn't satisfying when a wide region must be searched [10].

Another common approach in long code acquisition which does compress the uncertainty region is DAM [11]. It's similar to ZP method but has one more step of averaging before zero padding, in this way the parallel searching ability enhances by a factor of averaging length. It spends much less time than ZP and GZP for acquisition but on the other hand Doppler would affect the detection probability more. Generalized DAM (GDAM) is the application of GZP on DAM, which slightly improves the mean acquisition time and reduces the adverse Doppler effect on acquisition performance. DAM method is sensitive to phase offset between received and local code and the worst case occurs in $W/2$ phase offset, (W is the averaging length) [11, 12]. Therefore OAM is proposed to cope with this problem by adding shifted versions of the code to itself before averaging [11]. When there is no code phase offset, DAM and OAM almost perform the same, anyhow the offset is unknown and OAM would be more reliable hence. There's also generalized version of OAM (GOAM) that uses GZP for the zero padding part of the algorithm. A generalized form of averaging method (GAM) is proposed in [12], where detection performance and mean acquisition time has been analytically discussed, although deriving equations for DAM and OAM from the generalized form is not that easy and some deductions are vague, so in this paper a different method is underwent to analyze the averaging method. Applying DAM algorithm, WN samples of local code and received signal are taken where N is the FFT length. Averaging every W samples yields a block of N samples; afterwards the steps are the same as either ZP or GZP method, whichever chosen. If correlation result didn't pass the threshold, local code is shifted $WN/2$ samples and the same steps are followed. For OAM, the WN samples block of received code is added to its W samples shifted version before averaging. For local code, two shifted versions are added to the main WN samples of observation window; an $W/2$ samples shifted version multiplied by 2 and an W samples shifted version. The rest are the same as DAM except for that the code phase resolution of coarse acquisition in DAM is W samples whereas in OAM it's $2W$ samples because of shifted versions addition.

XFAST or extended replica folding acquisition search technique deals with vast uncertainty region in another way; this method adds blocks of local code of the same size together term by term (folding) reducing the size of primary observation window before correlation. If M is the number of folded blocks of local code, then the parallel searching capability is $(M-1)N+1$ that is better than averaging method, but the detection performance decays [9]. Dual Folding method copes with this by folding the received signal samples the same way as local code samples

This increases coherent integration time which leads to better detection performance that can get close to ZP by appropriate choice of folding numbers of received and local code samples, FFT length and etc. Increased coherent integration time however weakens the algorithm resistance against Doppler. DF method takes KN samples of received signal and folds them into N samples. On the local side, MN samples are observed and folded to N samples then the process of frequency domain correlation and threshold checking follows, if the detection is unsuccessful, local code will be shifted $(M-K)N$ samples and searched for offset again.

There are other approaches found in the literature based on these main methods that improve their performances in some aspects. These methods have mainly focused on coarse acquisition where the search region is wide, in [17] some techniques are proposed for fine acquisition which is needed after coarse acquisition. In [18] a method is proposed to improve ZP method by using the other half of correlation result which isn't used in traditional ZP method. In [19] a three step method is proposed where multiple code phases and frequency cells are combined to reduce the search space, this method employs ZP and XFAST to search between combined cells and then to refine the results. In [20] XFAST and DAM are both used to increase the speed of acquisition and to compensate for Doppler frequency, circular shift of FFT results is applied. In [21] and [22] a similar technique called divided folding is applied to improve the speed and Doppler compensation resolution, in both papers, the low detection probability of XFAST is not considered. In [23] an improved version of DAM is introduced where shifting and then averaging the local code is performed to enhance the detection probability. In [24] using GZP method for acquisition, the length of padded zeros is modified to enhance the detection probability and for alleviating the effect of jammer, first derivative of correlator is used. In [25] a modified version of ZP is proposed that increases the speed of acquisition at the cost of lower detection probability. In [26] and [27] dual channel method is proposed where zero padding method is applied in two parallel channels and uses the position of correlation peaks to verify the results, this method is not as fast as DF and so not useful for long code acquisition, [28] combines DC method with DF method to exploit speed and accuracy of both methods. Zero padding method is used beside DF method to boost Doppler tolerance of DF method in [29]. Extending coherent integration time (CIT) is always a good solution to increase detection probability in low SNR conditions, recently more researches are dedicated to recognize bit transitions and expand CIT further [30-34]. However this technique doesn't work for long code acquisition since the long code is not repeated in every bit or few bits.

3 DETECTION PERFORMANCE IN AWGN CHANNEL

In this section detection performance of mentioned main methods will be examined. Hereon, the detection probability of code phase offset which is dependent on various parameters like SNR is considered as detection performance. It is a good measure of system's ability to achieve synchronization. For the two fundamental averaging based algorithms i.e. DAM and OAM, probability of detection can be derived from those of GAM's [12], however the procedure is not descriptive enough and some ambiguities exist. Here probability of detection will be inferred for DAM and OAM specifically and are different from GAM's derivations.

The k-th sample of the In-phase (I) and Quadrature phase (Q) components for the received digital baseband signal of AWGN channel are as follows.

$$s_I[k] = Ad[k+\tau]c[k+\tau]\cos(\omega_D k\Delta t + \varphi) + n_I[k] \tag{1}$$

$$s_Q[k] = Ad[k+\tau]c[k+\tau]\sin(\omega_D k\Delta t + \varphi) + n_Q[k] \tag{2}$$

In which A is the signal amplitude, d is the modulated data bit $\in \{-1,+1\}$, c is the spreading code's chip $\in \{-1,+1\}$, ω_D is the residual frequency in radians, Δt is the sampling period, φ is the unknown carrier phase offset and n_I and n_Q each are additive white Gaussian noises for in-phase and quadrature phase components with zero mean and variance σ^2, $\mathcal{N}(0,\sigma^2)$.

3.1 Averaging

Using maximum likelihood (ML) criterion for unknown parameter estimation, The first step in acquiring detection probability is the correlation of the received signal with a code phase offset of τ and locally generated spreading code with offset of δ. Averaged received signal, $s_a[l]$, $l=\{0,1,2,\ldots\}$ will be:

$$s_a[l] = \sum_{k=0}^{W-1} s_I[k+lW] \tag{3}$$

Where W is the averaging length. Having NW samples, averaging yields a block of N samples, where N is the length of FFT. Correlation for the in-phase received signal and local code follows:

$$C_I[\delta] = \sum_{l=0}^{N-1} c[l+\delta] s_I[l] \tag{4}$$

3.1.1 Direct Average Method (DAM)

In DAM, averaging applies to both received signal and local code:

$$C_I[\delta] = \sum_{l=0}^{N-1}\sum_{m=0}^{W-1} c[m+lW+\delta] \sum_{k=0}^{W-1} s_I[k+lW] \tag{5}$$

According to DAM algorithm, after averaging the process is the same as GZP, considering the general form, GDAM, L zeros are padded to $N-L$ averaged samples of s_I and C_I changes to:

$$C_I[\delta-\tau]=\sum_{l=0}^{N-L-1}\sum_{m=0}^{W-1}\sum_{k=0}^{W-1}s_I[k+lW]c[m+lW+\delta]=$$

$$\sum_{l=0}^{N-L-1}\sum_{m=0}^{W-1}\sum_{k=0}^{W-1}Ad[\tau]c[k+lW+\tau]c[m+lW+\delta]\cos(\omega_D(k+lW)\Delta t+\varphi)+\sum_{l=0}^{N-L-1}\sum_{m=0}^{W-1}\sum_{k=0}^{W-1}n_I[k+lW]c[m+lW+\delta] \quad (6)$$

It's assumed that data bit, $d[\tau]$ is constant through the correlation (likely in most cases) and is replaced by $d[\tau]$. For the second term of (6), since AWGN is independent of spreading code, it has a distribution of $\sim\mathcal{N}(0,\ \sigma^2(N-L)W^2)$. In the first term of (6) two items exist: self-noise and coherent integration (CI) [9]. For true hypothesis, H_1, observation window of local code covers received signal partially such that n samples align with each other. Similar to analysis of DF and GZP in [6,5], for GDAM length of CI , n, is $(N-L)W$ so the mean value of CI part of the correlation, Γ_I , follows:

$$E_{\Gamma_I}=Ad[\tau]R_i(p)\sum_{l=0}^{W(N-L)}\cos(\omega_D l\Delta t+\acute{\varphi}) \quad (7)$$

Where $\acute{\varphi}$ is the phase of received signal at the first alignment with local code, $R_i(p)=\begin{cases}1-|p|, & H_1\\ 0, & H_0\end{cases}$ is the chip waveform correlation for true and false hypothesis. The residual code phase offset between the unaveraged samples of received signal and observation window of H_1 is denoted as θ and equals to pW where p is $\delta-\tau-\lfloor\delta-\tau\rfloor$, the residual code phase offset of averaged samples. θ shows how the performance of DAM deteriorates, it takes values between $-\frac{W}{2}$ and $+\frac{W}{2}$ so the worst case happens when the offset is half the averaging length which leads to half the correlation peak loss and is linear up to that point according to $R_i(p)$.

A pessimistic approximation of the variance of Γ_I is $A^2G_i(p)n$ where $G_i(p)=\begin{cases}p^2, & H_1\\ (1-|p|)^2, & H_0\end{cases}$. Thus the CI item has a distribution of $\sim\mathcal{N}(E_{\Gamma_I},\ \sigma^2{}_{\Gamma_I})$. For the self-noise part, since there's no overlaying it can be said that hypothesis H_0 is true. It is also modeled as a Gaussian random variable with zero mean, and the variance will be obtained as the CI's for the remaining samples; $W^2(N-L)-n$. Considering the worst case; $p=0$, the self-noise has a distribution of $\sim\mathcal{N}(0, A^2[(N-L)W^2-n])$.

So the correlation of In-phase received signal will be the sum of three Gaussian random variables i.e. AWGN, coherent integration and self-noise. The same result holds for Quadrature correlation as well, except for the mean value of the CI, where cosine function is replaced by sine. The envelope of the correlation result is $\Psi=\sqrt{C_I^2+C_Q^2}$ and has Rice distribution. So the probability density function conditioned on θ and H_i will be:

$$f_\Psi(x|\theta,H_i)=\frac{x}{\sigma_{H_i}^2}\exp\left\{-\frac{x^2+\mu_i^2}{2\sigma_{H_i}^2}\right\}I_0\left(\frac{x\mu_i}{\sigma_{H_i}^2}\right),\quad x\geq 0, i=0,1 \quad (8)$$

Where i indicates whether the observation window contains received signal and true hypothesis, H_1, holds or false hypothesis, H_0, holds that means there is no overlapping and $n=0$. I_0 is the zeroth order modified Bessel function of the first kind. μ_i^2 is the sum of squared means of I and Q random variables and the variance is:

$$\sigma_{H_i}^2=\sigma^2W^2(N-L)+A^2G_i(p)n+A^2(W^2(N-L)-n) \quad (9)$$

Detection probability can be considered either for whole observation block trueness or a single cell, in the latter case following Neyman-Pearson method, for a given false alarm probability, P_{fa}, the threshold can be derived from $P_{fa}=\int_{V_t}^{+\infty}f_\Psi(x|\theta,H_0)dx$ as $V_t=\sigma_{H_0}\sqrt{-2lnP_{fa}}$ therefore probability of detection for a single cell is obtained as:

$$P_{d,\theta}=Q_1\left(\frac{\mu_1}{\sigma_{H_1}},\frac{V_t}{\sigma_{H_1}}\right) \quad (10)$$

Where Q_1 is Marcum's Q function and

$$\mu_1^2=A^2\left(1-\left|\frac{\theta}{W}\right|\right)^2 sinc^2\left(\frac{\omega_D}{2}\Delta t(N-L)W\right)W^2n^2 \quad (11)$$

3.1.2 Overlap Average Method (OAM)

In OAM, the effect of unaveraged offset, θ, is mitigated by adding shifted versions to the received signal and local code. The correlation will be between received signal as: $s[k]+s[k+\frac{W}{2}]$ and local code as: $c[m]+2c\left[m+\frac{W}{2}\right]+c[m+W]$. The same process applies to OAM such that for every pair in correlation there are three Gaussian random variables as described for DAM; AWGN, CI and self-noise whose means and variances must be regained. There will be six terms in the result of in-phase correlation that are examined separately. The first term is the same as DAM, so for the in-phase correlation of $s[k]c[m]$ we have:

$$\mathcal{N}(0,\sigma^2(N-L)W^2) + \mathcal{N}\left(Ad[\tau]R_i(p)\sum_{l=0}^{W(N-L)}\cos(\omega_D l\Delta t + \varphi), A^2G_i(p)\big(W(N-L)\big)\right) + \mathcal{N}\big(0, A^2(W^2(N-L)-n)\big) \quad (12)$$

In which $p=\frac{\theta}{W}$ and $n=W(N-L)$. For the next product, $2s[k]c\left[m+\frac{W}{2}\right]$, the correlation is like (12) with these changes applied: variances of all Gaussians are multiplied by 4, mean value of second Gaussian is multiplied by 2. Due to $\frac{W}{2}$ shift, the averaged residual offset becomes $p=\frac{(\theta-\frac{W}{2})}{W}$. For $s[k]c[m+W]$, the correlation is exactly as (12) with $p=\frac{(\theta-W)}{W}$. The correlation of $s\left[k+\frac{W}{2}\right]c[m]$ is (12) with $p=\frac{(\theta+\frac{W}{2})}{W}$. For $2s\left[k+\frac{W}{2}\right]c\left[m+\frac{W}{2}\right]$ correlation is the same as of $2s[k]c\left[m+\frac{W}{2}\right]$ but with $p=\frac{\theta}{W}$. And for the last product, $s\left[k+\frac{W}{2}\right]c[m+W]$, the correlation is (12) with $p=\frac{(\theta-\frac{W}{2})}{W}$. Adding up all these random variables gives in a Gaussian random variable for In-phase correlation with the mean value of E_{Φ_I} which is the addition of means of all the mentioned Gaussians. For Q-phase correlation the variance is the same but in the mean value, E_{Φ_Q}, cosine is replaced with sine. The envelope follows Rice distribution as (8) where $\sigma_{H_i}^2$ is the summation of variances of depicted nine Gaussians. The other parameter, μ_i^2 is ${E_{\Phi_I}}^2+{E_{\Phi_Q}}^2$ which can be approximated by:

$$\mu_i^2 \approx n^2A^2\beta^2\left[3R_i\left(\frac{\theta}{W}\right)+3R_i\left(\frac{\theta-\frac{W}{2}}{W}\right)+R_i\left(\frac{\theta-W}{W}\right)+R_i\left(\frac{\theta+\frac{W}{2}}{W}\right)\right]^2 \quad (13)$$

Where $\beta^2=sinc^2(\frac{\omega_D}{2}n\Delta t)$ represents the loss due to residual carrier. Finally the probability of detection will be as (10) with updated values of μ_i^2 and $\sigma_{H_i}^2$.

3.2 Folding

The probability of detection for a single cell by Dual Folding method is derived in [9], it's in the form of (10) but conditioned on n and p with following parameters:

$$V_t=\sqrt{-2KMN(\sigma^2+A^2)\ln P_{fa}} \quad (14)$$

$$\mu_1^2=n^2A^2\beta^2(1-|p|)^2 \quad (15)$$

$$\sigma_{H_1}^2=KMN(\sigma^2+A^2)+np^2A^2-nA^2 \quad (16)$$

Where K and M are the folding numbers of the received signal and local code respectively. Setting $K=1$ will give the results for XFAST.

3.3 Zero Padding

For GZP method, probability of detection is derived almost the same way as DF except there's no self-noise item, so it has the form of (10) and conditioned on p with these parameters:

$$V_t=\sqrt{-2\ln P_{fa}(A^2(1-|p|)^2+\sigma^2)(N-L)} \quad (17)$$

$$\mu_1^2=(N-L)^2A^2sinc^2(\frac{\omega_D}{2}(N-L)\Delta t)(1-|p|)^2 \quad (18)$$

$$\sigma_{H_1}^2=(\sigma^2+A^2p^2)(N-L) \quad (19)$$

Where L is the length of zeros padded to received signal. For ZP method, $L=N/2$ and N is the FFT length.

4 DETECTION PERFORMANCE IN RAYLEIGH CHANNEL

Performance analysis of long code acquisition methods is generally done in AWGN channel and the effect of multipath fading channels on this communication technique is not examined thoroughly. In this section the performance of aforementioned methods are analytically evaluated and compared with simulations in multipath fading channels.

The fading channel considered here, is composed of F_p independent slowly fading paths. AWGN of each path is denoted by $n_f(t)$. The impulse response of fth path of F_p paths is

$$h_f(t)=\alpha_f e^{j\theta_f}\delta(t-\tau_f) \quad (20)$$

In which $\{\alpha_f\}_{f=1}^{F_p}$, $\{\theta_f\}_{f=1}^{F_p}$, $\{\tau_f\}_{f=1}^{F_p}$ are amplitude, phase and delay in each path respectively that are imposed on transferred signal and are mutually independent. Since in this paper we consider slow fading, these random variables are constant through symbol duration, T_s. The first path is the reference path so its delay is $\tau_1 = 0$ and other delays are assumed ascending as $\tau_2 < \tau_3 < \cdots < \tau_{F_p}$. Also it's presumed that the carrier phase is zero and the receiver knows the beginning of the transmission. If the delay is unknown and random it's assumed that its distribution is uniform in $[0, T_s)$. For random variable of phase uniform distribution is assumed in the interval of $[0,2\pi)$. If the amplitude is considered as random, the distribution is Rayleigh [24]:

$$P_{a_f}(\alpha_f) = \frac{2\alpha_f}{\Omega_f}\exp\left(\frac{-\alpha_f^2}{\Omega_f}\right), \qquad for\ \alpha_f \geq 0 \tag{21}$$

In which $\Omega_f = E\{\alpha_f^2\}$. The in-phase component of the received baseband digital signal from fth fading path is

$$s_{I,f}[k] = A\alpha_f d[k+\tau_f]c[k+\tau_f]\cos(\omega_D k\Delta t + \theta_f) + n_{I,f}[k] \tag{22}$$

In this equation, $n_{I,f}[k]$ is the complex random process of AWGN of fth fading path with variance of σ_f^2 and is independent from other paths' AWGN. The equations mentioned for the correlation of received signal and local spreading code for GDAM method in fading channel changes accordingly,

$$C_{I,f} = \sum_{f=1}^{Fp}\sum_{l=0}^{L-N-1}\sum_{m=0}^{W-1}\sum_{k=0}^{W-1} A\alpha_f d[\tau_f]c[lW+k+\tau_f]\cos(\omega_D(k+lW)\Delta t + \theta_f) + \sum_{f=1}^{Fp}\sum_{l=0}^{L-N-1}\sum_{m=0}^{W-1}\sum_{k=0}^{W-1} n_{I,f}[k+lW]c[lW+\delta+m] \tag{23}$$

Similar to analysis of detection performance in AWGN channel, in fading channel the correlator's output is consisted of three items; the first item is the CI and the second is inherent noise which are explained earlier. The other part of correlator's output in (23) is the correlation of additive white noise with local code. Among these three parameters only the CI part is infected by fading and other items are derived as they were for AWGN channel.

For determining detection probability, the expectation and the variance of CI random variable should be calculated. If all the three parameters of fading channel are random, CI will be a function of these random variables and obtaining the expectation value needs calculating the triple integral in (24) where each integral is indeed a F_p fold integral.

$$E[CI] = \iiint CI(\alpha_f,\tau_f,\theta_f)P(\alpha_f)P(\tau_f)P(\theta_f)d\alpha_f d\tau_f d\theta_f \tag{24}$$

Depending on the channel state information, different cases may occur in counting fading effect. If sufficient knowledge of channel parameters is provided, applying the decision metric derived from CI random variable yields the optimum receiver for fading channel that is generally called RAKE receiver [35]. Since the optimum receiver is not yet designed for long code acquisition in fading channels, here the effect of two cases of fading channel will be studied on long code acquisition. The receiver is like an AWGN channel receiver which calculates the correlation of received signal with local code.

4.1 Unknown Amplitude, phase and delay

In this case, the amplitude of channel's impulse response has Rayleigh distribution. Phase and delay are uniformly distributed as mentioned earlier. Averaging on these three random variables in (23) gives the expectation of CI as

$$E[CI] = \frac{1}{T_s}\{A\alpha_f d[\tau_f]R_i(p)\sum_{l=0}^{W(L-N)}\cos(\omega_D l\Delta t + \theta_f)\} \tag{25}$$

In this statement, averaging on uniform random variable of phase, results in zero expectation for CI and this technically eliminates the possibility of detection. This also applies to other long code acquisition methods.

4.2 Unknown Amplitude, known phase and delay

In this case, the impulse response of the multipath fading channel has a Rayleigh distributed random amplitude whereas the phase and delay are known. Again, the statistics of CI random variable must be calculated. The expectation and variance of CI for fth path of fading channel are as follows,

$$E[CI] = Ad[\tau_f]R_i(p)\sum_{l=0}^{W(L-N)}\cos(\omega_D l\Delta t + \theta_f)\int \alpha_f P_{a_f}(\alpha_f)d\,\alpha_f \tag{26}$$

Where the integral is the expectation of Rayleigh distribution, $\int \alpha_f P_{a_f}(\alpha_f)d\,\alpha_f = \sqrt{\frac{\pi\Omega_f}{4}}$. Variance of CI is derived accordingly wherein the integral is calculated using equation 3.461.3 [36].

$$var[CI] = A^2 G_i(p) \int \alpha_f^2 P_{a_f}(\alpha_f) d\,\alpha_f = \frac{\Omega_f^2 A^2 G_i(p)}{2} \quad (27)$$

These three Gaussian random variables which are white noise, inherent noise and CI (with derived statistics) compose correlator's output for other fading paths. Putting together the output of all these paths yields a Gaussian random variable for correlation result of in-phase received signal and local code whose mean and variance are the sum of means and variances of those three Gaussian variables. For determining detection probability the process is the same as it was for AWGN channel. The random variable of detector output has Rice distribution with following parameters

$$\mu_1^2 = \sum_{f=1}^{F_p} A^2 \left(1 - \left|\frac{\theta}{W}\right|\right)^2 sinc^2\left(\frac{\omega_D}{2}\Delta t(L - N)W\right) W^2 n^2 \frac{\pi \Omega_f}{4} \quad (28)$$

$$\sigma_{H_i}^2 = \sum_{f=1}^{F_p} \sigma_f^2 W^2 (L-N) + \frac{\Omega_f^2 A^2 G_i(p)}{2} + A^2(W^2(L-N) - n) \quad (29)$$

Eventually the detection probability of GDAM for multipath fading channel is obtained from (10) substituting above parameters. The effect of fading channel on detection probability of GOAM is obtained similar to GDAM. The expressions of mean and variance of GOAM is the same as GDAM. Using DF method in Rayleigh channels changes the parameters of detection probability as below

$$V_t = \sqrt{\sum_{f=1}^{F_p} -2KML\left(\sigma_f^2 + A^2\right) \ln P_{fa}} \quad (30)$$

$$\mu_1^2 = \sum_{f=1}^{F_p} n^2 A^2 \beta^2 (1 - |p|)^2 \frac{\pi \Omega_f}{4} \quad (31)$$

$$\sigma_{H_1}^2 = \sum_{f=1}^{F_p} KML\left(\sigma_f^2 + A^2\right) + \frac{n p^2 A^2 \Omega_f^2}{2} - nA^2 \quad (32)$$

In which $K = 1$ yields the performance of XFAST method for Rayleigh channel. In GZP method, the same process of obtaining detection probability for GDAM is followed and detection probability parameters changes as following

$$V_t = \sqrt{\sum_{f=1}^{F_p} -2 \ln P_{fa} (A^2 (1 - |p|)^2 + \sigma_f^2)(L - N)} \quad (33)$$

$$\mu_1^2 = \sum_{f=1}^{F_p} (L-N)^2 A^2 sinc^2 (\frac{\omega_D}{2}(L-N)\Delta t)\,(1 - |p|)^2 \frac{\pi \Omega_f}{4} \quad (34)$$

$$\sigma_{H_1}^2 = \sum_{f=1}^{F_p} \left(\sigma_f^2 + \frac{A^2 p^2 \Omega_f^2}{2}\right)(L-N) \quad (35)$$

5 ACQUISITION TIME

Another important criterion in the assessment of an acquisition algorithm, especially for long codes, is the time consumed for detecting the code phase, which is defined as an expected value. A general approach is described in [37] that can be used for long codes either. Figure 2 shows the flow graph diagram of a typical acquisition method used for obtaining acquisition time. Main portions of the diagram which differs between algorithms are the number of observation windows, Λ, false alarm penalty, γ, and number of states in which true hypothesis, H_1, may occur. For example in DF method there are multiple states that may lead to acquisition. In figure 2 only one state leads to acquisition since in most methods this is the case that happens. $\{\pi_i\}$ is the priori distribution of states that is considered as uniform one hereafter. Mean acquisition time for the serial search can be considered as the general form:

$$E(T_{ACQ}) = \frac{T_D}{P_d^b}\left\{1 + (1 + \gamma P_{fa}^b)(2 - P_d^b)\frac{(\Lambda - 1)}{2}\right\} \quad (36)$$

Where P_d^b and P_{fa}^b are detection and false alarm probability for the block of observation window respectively. T_D is the dwell time for searching an observation window. Λ is the number of observation windows in an uncertainty region of Θ code phases.

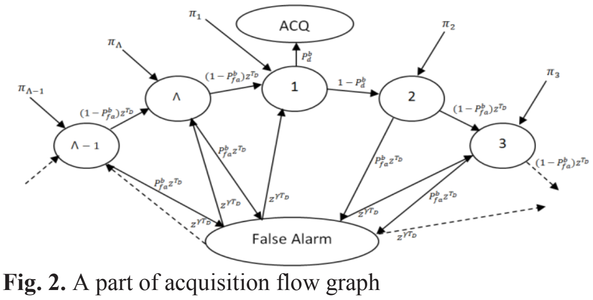

Fig. 2. A part of acquisition flow graph

5.1 Averaging

Following Eq. (20) and similar analysis in [5] and [6], for DAM and OAM method, $\gamma = W$ since no averaging would be applied for false alarm checking. Number of observation windows is $\Lambda = \left\lceil \frac{2\Theta}{WN} \right\rceil$. $\lceil . \rceil$ gives the smallest integer number bigger than the input). The block false alarm probability is $P_{fa}^{b} = 1 - (1 - P_{fa})^{L}$ since after zero padding, only the first L results are preserved, P_{fa} is the desired false alarm probability. And the block detection probability is $P_{d}^{b} = 1 - (1 - P_{fa})^{L-1}(1 - P_{d,\theta})$ where $P_{d,\theta}$ is the corresponding detection probability of DAM or OAM with unaveraged offset of θ.

5.2 Folding

The descriptive analysis for mean acquisition time for DF method is provided in [9], which takes into account different states of acquisition leading to an extensive equation. Here we make some assumptions, simplifying and realizing the mean acquisition time. We presume the acquisition level only contains one state, in which the whole incoming signal is covered by the observation window, so the coherent integration length will be $n = KN$. The presumption makes the mean acquisition time independent of the offset between the first elements of incoming signal and local code. Also we take the pessimistic assumption of $p = 0$ meaning there's no residual offset. Consequently the mean acquisition time of DF reduces to (20) with following parameters:

$$\Lambda = \left\lceil \frac{\Theta}{(M-K)N+1} \right\rceil \tag{37}$$

$$P_{d}^{b} = 1 - (1 - P_{fa})^{N-1}(1 - P_{d|n=KN,p=0}) \tag{38}$$

$$P_{fa}^{b} = 1 - (1 - P_{fa})^{N} \tag{39}$$

$\gamma = M$ is the false alarm punishment of DF, which means for testing the detected observation window, no folding is applied. Setting $K = 1$ yields the mean acquisition time for XFAST.

5.3 Zero Padding

For GZP mean acquisition time is obtained through (20) with $\gamma = 1$ and following parameters [10]:

$$\Lambda = \left\lceil \frac{\Theta}{L+1} \right\rceil \tag{40}$$

$$P_{fa}^{b} = 1 - (1 - P_{fa})^{L} \tag{41}$$

$$P_{d}^{b} = 1 - (1 - P_{fa})^{L-1}(1 - P_{d}) \tag{42}$$

Setting $L = N/2$ yields the result for ZP method. P_d is the detection probability of GZP or ZP.

6 RESULTS AND DISCUSSIONS

In this section, all the introduced methods are simulated for some conditions and the results are depicted to provide a prospect of each method's performance in comparison to each other. A maximal sequence generated by a 24-stage Linear Feedback Shift Register (LFSR) is chosen as the long spreading code. The sampling frequency is the same as the chip rate of GPS P-code i.e. 10.23 MHz. False alarm probability is set as 10^{-5}. The number of code phases in the uncertainty region is $\Theta = 10^{7}$ and FFT length is the popular 1024. Figure 3 shows the analytical and simulation results for the detection probability of long code coarse acquisition techniques. XFAST with folding parameter of $M = 20$ is the weakest, while its improved version, DF $(M = 35, K = 16)$ performs much better close to GZP. Coherent integration length in folding methods is set pessimistically as $n = KN$. The variant number of padded zeros in GZP is set as a quarter of FFT length according to [10] which yields the best detection performance preceding ZP. The code phase offset is chosen the way that θ is half the averaging length, W, which is the worst case for averaging methods. In Generalized DAM (GDAM) and Generalized OAM (GOAM), padded zeros is chosen as GZP, $L = N/4$. For averaging methods, different averaging lengths like 128,256,512 gives the same results that in their best case resembles ZP. But for the chosen code phase offset, simulation confirms the success of OAM in coping with the deficiency of DAM. Here generalized versions of both averaging methods are simulated which are app. 2dB better than the original methods.

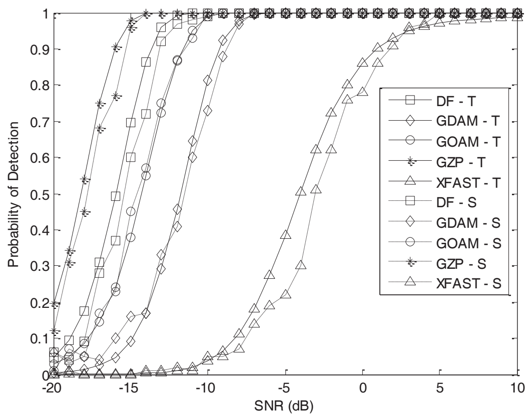

Fig. 3. Probability of Detection for long code acquisition methods, residual frequency is zero - Theoretical (solid line, 'T'), Simulation (dashed line, 'S').

The effect of Doppler appears as $sinc^{2}(\frac{\omega_D}{2\pi} n\Delta t)$ in detection probability where n is the coherent integration length. The first zero-crossing of this function happens in $f_D = \frac{1}{n\Delta t}$ and according to the curve of $sinc^{2}(.)$ function a Doppler frequency between half or third of this value is the Doppler frequency tolerable by correlator, meaning that this amount of frequency has trivial effect on detection probability so it can be chosen as the width of frequency search bin. Residual frequency of Doppler effect depends on carrier frequency and rational speed between transmitter and receiver. Figure 4 shows the probability of detection for averaging methods for residual frequency of 150 Hertz which causes a decrease about 4-5 dB in their performances. Meanwhile for DF, XFAST and GZP even the residual frequency of 1000 Hertz is tolerable as shown in Figure 5. This difference is obviously due to n, such in averaging methods larger coherent integration time leads to better detection probability but since Doppler tolerance is decreased frequency search bins get smaller. In case of large residual frequency averaging methods take longer to search for frequency than DF, XFAST and GZP.

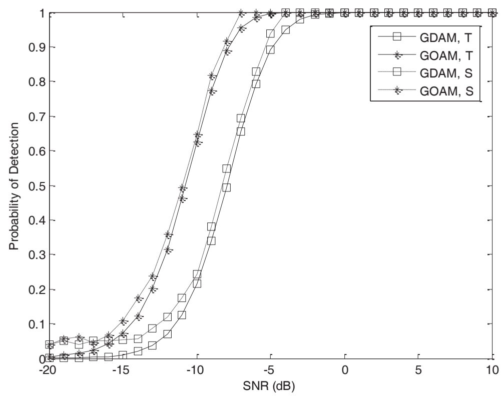

Fig. 4. Probability of detection for GDAM and GOAM, residual frequency is 150 Hz - Theoretical (solid line, 'T'), Simulation (dashed line, 'S')

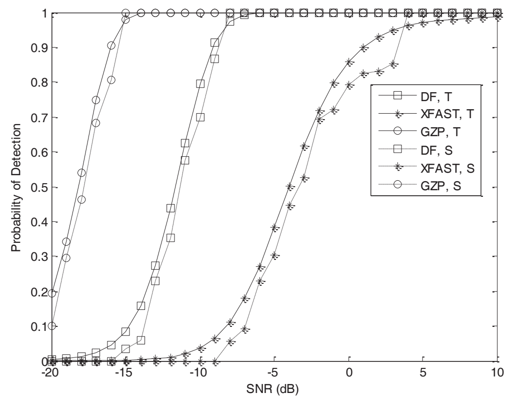

Fig. 5. Probability of Detection for GZP, XFAST and DF, residual frequency is 1000 Hz - Theoretical (solid line, 'T'), Simulation (dashed line, 'S')

Investigating the effect of FFT block length is depicted in Figure 6. For mentioned methods considering $P_d \cong 0.9$ there's almost $3-4\,dB$ improvement in detection performance as the block length is expanded to 2048. Figure 3-6 displays good agreement between theoretical and simulation results especially for the derived equations of GOAM.

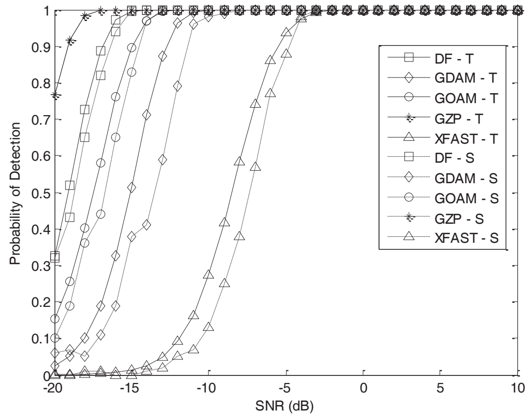

Fig. 6. Probability of Detection for long code acquisition methods, FFT length is L=2048 - Theoretical (solid line, 'T'), Simulation (dashed line, 'S').

For simulating multipath fading channel effect on detection performance, a slowly non-frequency selective fading channel consisting of three paths with known delay and phase and Rayleigh distributed amplitude is considered. The average gains of paths are $0\,dB, -3\,dB, -5\,dB$ for first, second and third path respectively. Maximum Doppler shift of the channel is assumed as 30 Hz. Also it is assumed that the additive white noises of all paths are all the same, $\sigma_f^2 = \sigma^2$. The detection probability of GDAM with averaging length of $W = 128$ with same simulation conditions as AWGN channel is shown in Figure 7. Comparing with AWGN channel, the performance decreases $5\,dB$.

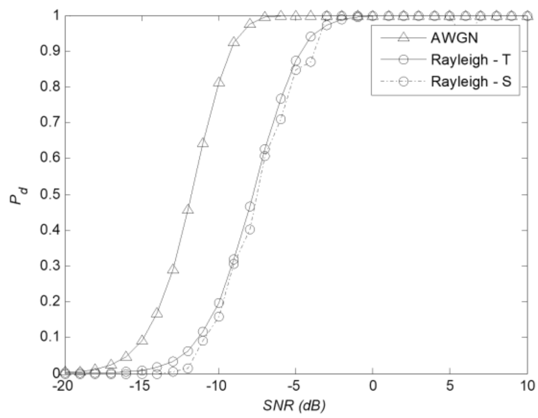

Fig. 7. Detection probability of GDAM in a three-path Rayleigh channel vs. AWGN channel - Theoretical (solid line, 'T'), Simulation (dashed line, 'S').

Detection probability of GOAM in mentioned Rayleigh channel shows the same behaviour as GDAM and simulations shows $4\,dB$ decrease in detection performance. Figure 8 shows $4\,dB$ decrease in DF method detection performance.

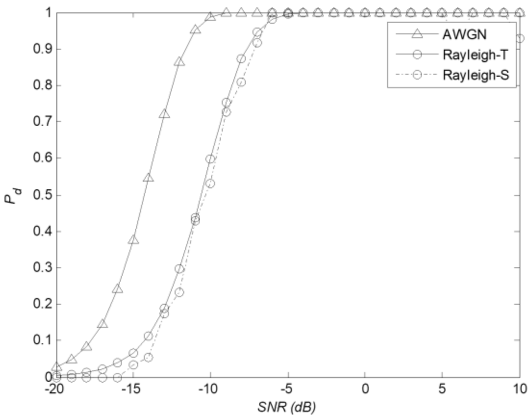

Fig. 8. Detection probability of DF in a three-path Rayleigh channel vs. AWGN channel - Theoretical (solid line, 'T'), Simulation (dashed line, 'S').

Figure 9 shows detection probability of GZP method in both Rayleigh and AWGN channels. 3 dB decrease in performance is observed if the channel is multipath faded.

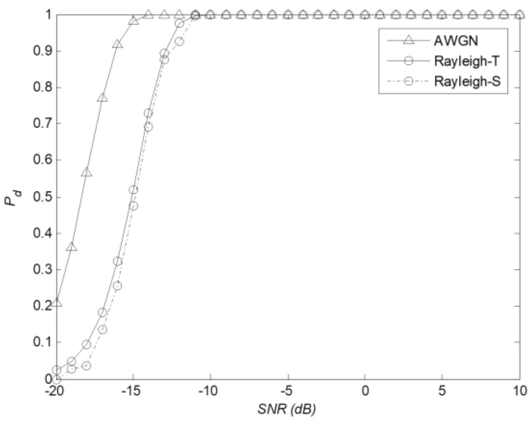

Fig. 9. Detection probability of GZP in a three-path Rayleigh channel vs. AWGN channel - Theoretical (solid line, 'T'), Simulation (dashed line, 'S').

In depicted figures good agreement between theoretical results and simulation is observed. These results also show that GZP method is slightly more resistant against fading than other methods. It can be deduced that passing through a multipath fading channel causes $3-5$ dB decrease in detection performance.

Figure 10 shows the mean acquisition time as a factor of T_D. The graphs are produced using (36) and theoretically calculated detection probabilities. Although GZP is the best in detection performance but in time consumption it is the worst. XFAST is somehow better than GZP but in low SNR which is the case most often, GZP is faster. There is a close trade-off between DF, GOAM and GDAM. DF is better in detection probability, while the two latter are better in mean acquisition time. Depending on which parameter is more critical, the most appropriate method can be chosen.

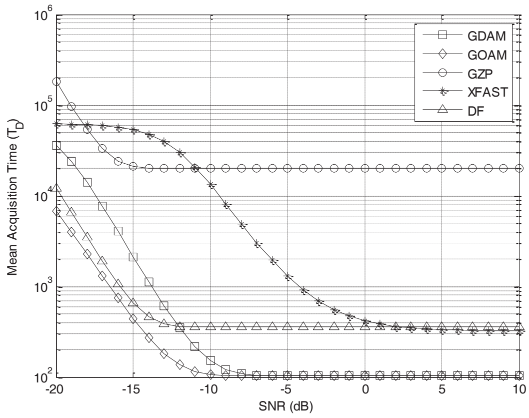

Fig. 10. Mean Acquisition Time of long code acquisition methods versus SNR for AWGN channel

Another effective factor in picking out a method is the resource consumption and implementation. Here this factor is presented as floating point operations (FLOPS) each method performs in every dwell including fine and coarse acquisition in given SNR. The numbers are obtained through estimated usage of MATLAB software and corresponding processor performance so may differ a little for every processor. This comparison is rough since one may write a different script for an algorithm. It shows that GZP and GOAM consume more computational resources while DF is the best among the others. Table 1 provides a general comparison between methods including FLOPS of each algorithm.

Algorithm		Folding		Zero Padding	Averaging	
		XFAST	DF	GZP	GOAM	GDAM
SNR ($P_d \cong 1$)		$> 4dB$	$> -12dB$	$> -14dB$	$> -10dB$	$> -7dB$
Shortest MAT	$SNR \leq -10dB$	5 (worst)	2	4	1 (best)	3
	$SNR > -10dB$	3	2	4 (worst)	1 (best)	1 (best)
Doppler tolerance		good		good	bad	
Giga FLOPS		1.07	0.3	6.8	8.03	1.9

Table 1. A comparison between long code acquisition methods for AWGN channel.

7 CONCLUSION

In this paper, common methods of acquisition in long spreading code DSSS were introduced beside their algorithms. Theoretical equations of each method were described by same notation in terms of probability of detection and mean acquisition time; meanwhile two main averaging methods of DAM and OAM are theoretically analysed and expressions for detection and acquisition time are derived. Also the methods are theoretically examined in multipath fading channel with Rayleigh distribution and detection probability expressions are derived.

Detection performance and time consumption of the methods are compared in AWGN and Rayleigh fading channels as well as the effect of Doppler and coherent integration time. Computational burden of each algorithm is obtained as floating point operations performed for each run of the coarse and fine acquisition. Simulations results in compliance with theories, show better detection performance of GZP with the cost of more time and resource consumption. According to the results, in the case of Doppler effect, averaging methods endure less residual frequency than folding and zero padding methods. Doubling FFT block length improves detection of all methods to the same degree. Rayleigh fading causes detection performance degradation of few dBs while GZP method is less affected. Mean acquisition time of GZP is the highest but in low SNRs, XFAST is the slowest. DF, GOAM and GDAM time consumption is close to each other and is better than XFAST and GZP. Regarding computational resource consumption, DF is the lightest algorithm while GOAM consumes resources more than the others. In conclusion, DF seems to present a better trade-off between detection performance, acquisition time and resource management.

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